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Optimal Commutation Law for Three-Phase Surface-Mounted Permanent Magnet Linear Synchronous Motors

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Abstract— **Sinusoidal commutation of phase currents leads to force ripple, if the motor back-EMFs differ from ideal case. Force ripple reduces the tracking performance significantly, if no compensation methods are applied. This paper presents a method to optimize the commutation in order to prevent force ripple. No assumptions of periodicity, symmetry, shape or balance of the motor back-EMFs are made. The proposed commutation law considers nonidealities of motor and amplifier. It is based on non-parametric force functions, which are identified in a proposed procedure. The optimal commutated currents produce minimal winding losses and therefore maximize motor efficiency. The commutation law is valid for any velocity and any desired thrust force. In this research, three-phase synchronous motors with surface-mounted magnets are considered. Experiments are performed with a linear motor, but the results are also valid for rotating motors.**

I. Introduction

Permanent magnet (PM) linear synchronous motors (LSM) are mainly used in applications which require high speed and high precision in positioning such as semiconductor and electronic manufacturing or laser cutting. PM LSMs have better performance and higher thrust density than linear induction motors [1]. The main benefits of PM LSMs are the high force density achievable and the high positioning precision and accuracy associated with the mechanical simplicity of such systems. The electromagnetic force is applied directly to the payload without any mechanical transmission such as chains or screw couplings. Today's state-of-the-art linear motors can achieve velocities up to $10\,\mathrm{m/s}$ and accelerations of $25\,\mathrm{g}$ [2].

The more predominant nonlinear effects underlying a PM LSM system are friction and force ripple arising from imperfections in the underlying components [3]. There are two types of position dependent disturbances: cogging force and force ripple. Force ripple is an electro-magnetic effect and causes a periodic variation of the force constant. Only if the back-EMF waveforms are sinusoidal and balanced, symmetric sinusoidal commutation of the phase currents produces smooth force. Cogging force is a disturbance force that is independent of the motor currents.

In order to avoid force ripple, different methods have been developed. The motor design can be adapted to reduce force ripple, but this leads to a decrease in mean force [4]. In [5] several techniques of torque ripple minimization for rotating motors are reviewed. In [6] and [7] the Park transformation is extended, in order to prevent torque undulation. In [8] a force ripple model is developed and identification is carried out with a force sensor and a frictionless air bearing support of the motor carriage. In [9] a neuronal-network based feedforward controller is proposed to reduce the effect of force ripple. Position-triggered repetitive control is presented in [10]. Other approaches are based on disturbance observers [11], iterative learning control [3] or adaptive control [12].

In [13], a force ripple compensation method for PM LSM systems with electronic commutated servo amplifiers was presented. The parameters of the force ripple are identified in an offline procedure. The proposed method applies a feedforward controller for compensation of force ripple and friction. The force ripple is produced by the imperfections of the underlying components, particularly non-optimal commutation of phase currents in the servo amplifier. The drawback of controller-based compensation methods in conjunction with electronic sensor-based commutation is the non optimal current excitation of the motor. Imperfect commutation of the phase currents produces additional heat in the phase windings and therefore reduces the motor efficiency.

In this paper, a force ripple compensation method for software commutated servo amplifiers is proposed. The additional input signal of a software commutated amplifier is used to optimize the operation of the motor. The paper proposes a method for design of the phase current commands for minimization of force ripple and maximization of motor efficiency. The commutation of the phase currents is optimized in order to get smooth force and minimal winding losses. The identification of the force functions is performed by measuring the control signals in a closed position control loop. This paper extends the work, the author has presented in [14], [15], in several ways. Simple equations for calculation of the current commands are proposed and the problem of numerical optimization is overcome. Furthermore the paper removes some assumptions which are made in previous works. This paper is limited to three-phase motors. For motors with a larger number of phases, it is referred to [16].

II. Modeling of Three-Phase Servo Systems

In this section a model of force generation for PM LSMs is developed. A PM LSM consists of a secondary and a primary. The primary contains the phase windings, the secondary consists of rare earth magnets, mounted in alternate polarity on a steel plate. The electromagnetic thrust force is produced by the interaction between the secondary and the magnetic field in the primary driven by the phase currents of a servo amplifier. In PM LSMs, three different types of forces exists:

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- *Cogging force:* Cogging is a magnetic disturbance force that is caused by attraction between the PMs and the iron part of the primary. The force depends on the relative position of the primary with respect to the magnets, and it is independent of the motor current. The mean value of cogging is zero. Cogging is negligible in motors with iron-less primaries or slotless motor design [17]. Cogging can not be compensated by means of optimization of commutation, since it is independent of excitation currents. If cogging is significant, a position dependent signal has to be added to the force command to overcome the cogging force. This can be made for example by means of feedforward control [9].
- *Reluctance force:* Reluctance force occurs only in motors with interior-mounted PMs. In this type of motor, the reluctance of the motor is a function of position. The self inductance of the phase windings varies with position of the primary with respect to the secondary. When current flows, this causes a position dependent force. If PMs are surface-mounted, reluctance is constant and reluctance force is negligible [4]. In this work reluctance force will not be considered.
- *Excitation force:* Excitation force is produced by the interaction between the magnetic field in the secondary and the magnetic field of the phase windings. In Surface-mounted PM-motors, this is the dominant force production mechanism [18]. In this paper only this type of force is considered.

Excitation force is proportional to the magnetic field and the phase currents i_A, i_B, i_C. The back-EMF induced in a phase winding (e_A, e_B, e_C) is proportional to the magnetic field and the speed of the motor. The total thrust force F_{thrust} is the sum of the forces produced by all phases:

$$\dot{x}\,F_{\text{thrust}} = \sum_p e_p(x)\,i_p\,, \qquad p \in \{A, B, C\}. \tag{1}$$

The speed-normalized back-EMF waveforms $\frac{e_p(x)}{\dot{x}}$ can also be interpreted as the force functions of the phases ($K_{Mp}(x)$). With this force functions the thrust force can be rewritten as:

$$F_{\text{thrust}} = \sum_p K_{Mp}(x)\,i_p\,, \qquad p \in \{A, B, C\}. \tag{2}$$

Only if the force functions are sinusoidal and balanced, symmetric sinusoidal commutation of the phase currents produces smooth force. Force ripple occurs, if the motor current is different from zero, and its absolute value depends on the required thrust force and the relative position of the primary with respect to the secondary. There are several sources of force ripple in motor and amplifier. Due to production tolerances, phase and amplitude of back-EMFs may be imbalanced. Furthermore the shape of back-EMFs may differ from ideal sinusoidal shape, due to motor design. Since the amplifier drives the phase currents, it may also be a source of force ripple. Production tolerances of the amplifier may lead to offset currents, imbalance of current gains, and measurement gains.

Inspection of (2) shows, that offset currents lead to force ripple with the same period as the force functions. This force ripple is independent of the desired thrust force. Amplitude or phase imbalance of the motor and imbalance of amplifier gains lead to force ripple with half commutation period which scale in direct proportion to the desired thrust force. A k^{th} order harmonic of a back-EMF produces $(k-1)^{th}$ and $(k+1)^{th}$ order harmonic force ripple if a sinusoidal current is applied.

Force ripple with the same period as the commutation period is independent of the desired current, all higher order harmonics scale in direct proportion to the desired current because of the linearity of the force equation (2).

There are two different wirerings of three-phase PM-motors: independent phase currents or star-connection.

A. Motors with Independent Phase Currents

In this kind of motor, the phase currents are independent of each other. This operation requires a more expensive servo amplifier and wirering than a servo system with star-connection. The phase currents are driven by a servo amplifier which needs one command signal for each of the three phases. The phase currents are proportional to this signals

$$i_p \approx i_{p\text{ref}} = K_{Sp}\,u_p\,, \qquad p \in \{A, B, C\}, \tag{3}$$

where u_p are the current commands and K_{Sp} are the amplifier gains. In this paper the amplifier gain is defined as the conversion factor of the external amplifier input u and the internal reference current i_{ref} (cp. section V-A). The dynamics of the servo amplifier will not be considered as they play a minor role in the dynamics of the whole system.

The thrust force produced by the motor is a function of the current commands and the position

$$F_{\text{thrust}} = K_A(x)\,u_A + K_B(x)\,u_B + K_C(x)\,u_C, \tag{4}$$

where u_A, u_B, u_C are the current commands and $K_A(x)$, $K_B(x)$, $K_C(x)$ are position dependent force functions of the phases, which are given by

$$K_p(x) = K_{Sp}\,K_{Mp}(x)\,, \qquad p \in \{A, B, C\}. \tag{5}$$

Equation (4) depends linear on the current commands and is nonlinear in the position x. In sinusoidal commutation, sinusoidal force functions are assumed. The current commands are functions of the force command u and position x and are defined by

$$\begin{aligned}
u_A(u,x) &= \frac{2}{3}\,\sin(\vartheta(x))\,u, \qquad (6)\\
u_B(u,x) &= \frac{2}{3}\,\sin\left(\vartheta(x) \pm \frac{2\pi}{3}\right)u,\\
u_C(u,x) &= \frac{2}{3}\,\sin\left(\vartheta(x) \mp \frac{2\pi}{3}\right)u,\\
\text{with}\quad \vartheta(x) &= \frac{\pi}{\tau_p}\,(x - x_0)\,,
\end{aligned}$$

where τ_p is the pole pitch and x_0 is the zero position with maximum force. The sign in $\sin\left(\vartheta(x) \pm \frac{2\pi}{3}\right)$ depends on the wirering of the motor. In case of sinusoidal commutation, the force equation can be rewritten as

$$\begin{aligned} F_{\text{thrust}} &= K_{F\sin}(x)\, u \quad \text{with} \qquad (7)\\ K_{F\sin}(x) &= \frac{2}{3}\Bigg(\sin(\vartheta(x))\ K_A(x) + \sin\left(\vartheta(x) \pm \frac{2\pi}{3}\right) K_B(x) + \\ &\qquad \sin\left(\vartheta(x) \mp \frac{2\pi}{3}\right) K_C(x)\Bigg). \end{aligned}$$

In the ideal case, the force functions $K_A(x)$, $K_B(x)$ and $K_C(x)$ are symmetric, balanced sinusoids, which leads to $K_{F\sin} \neq f(x)$. In this case, the force depends only on the force command u, and is independent of the position x. Usually, the force functions deviate more or less from the ideal case, and $K_{F\sin}$ is a function of position.

B. Motors with Star-Connection

The star connection is the most common configuration in three-phase motors [18]. The neutral of the star is not brought out. The three external lines of the star are connected to a half bridge circuit. The collection of three half bridges is called a three-phase bridge. Only six power electronic devices are needed in a three-phase bridge, this is minimal compared to any other number of phases. Only two of the three phase currents are independent, the third phase current depends on the others:

$$i_C = -i_A - i_B. \qquad (8)$$

The phase currents are driven by a servo amplifier which needs only two command signals for the three phases:

$$\begin{aligned} i_A &= K_{SA}\, u_A\,, \quad i_B = K_{SB}\, u_B\,, \qquad (9)\\ i_C &= -K_{SA}\, u_A - K_{SB}\, u_B. \end{aligned}$$

The force equation can be written as

$$\begin{aligned} F_{\text{thrust}} &= K_A(x)\, u_A + K_B(x)\, u_B \quad \text{where} \qquad (10)\\ K_A(x) &= K_{SA}\ (K_{MA}(x) - K_{MC}(x)) \quad \text{and}\\ K_B(x) &= K_{SB}\ (K_{MB}(x) - K_{MC}(x))\,. \end{aligned}$$

In sinusoidal commutation the current commands are chosen as follows:

$$\begin{aligned} u_A(u,x) &= \frac{2}{3}\ \sin(\vartheta(x))\ u, \qquad (11)\\ u_B(u,x) &= \frac{2}{3}\ \sin\left(\vartheta(x) \pm \frac{2\pi}{3}\right) u, \end{aligned}$$

which yields the thrust force as

$$\begin{aligned} F_{\text{thrust}} &= K_{F\sin}(x)\, u \quad \text{with} \qquad (12)\\ K_{F\sin}(x) &= \frac{2}{3}\Bigg(\sin(\vartheta(x))\ K_A(x) + \\ &\qquad \sin\left(\vartheta(x) \pm \frac{2\pi}{3}\right) K_B(x)\Bigg). \end{aligned}$$

Thrust force is only independent of position in the ideal case, where the force functions $K_A(x)$ and $K_B(x)$ are balanced sinusoids.

III. Optimal Commutation of Phase Currents

Aim of optimal commutation is to obtain current commands that generate a thrust force which depends only on the force command and which is independent of the position. Inspection of the force equations (4) and (10) reveals, that there are an infinite number of combinations of current commands that generate the desired force. Therefore a secondary condition has to be applied. In this research, the ohmic winding losses of the motor are minimized. In permanent magnet synchronous motors, the winding losses are the main source of power dissipation.

The problem is formulated as constrained optimization, where the thrust force is the constraint and the losses are minimized. For optimization purposes, the different wirerings have to take into account.

A. Motors with Independent Phase Currents

In case of independent phase currents the constraint can be described by

$$\begin{aligned} F_{\text{thrust}} &= \sum_p K_p(x)\, u_p\,, \qquad p \in \{A, B, C\}, \qquad (13)\\ &= K_F\, u \neq f(x), \end{aligned}$$

where K_F is a freely eligible constant and u is the force command. The winding losses are given by

$$P_{\text{cu}} = \sum_p R_p\, i_p^2\,, \qquad p \in \{A, B, C\}, \qquad (14)$$

where R_p is the resistance of phase winding p. In assumption of symmetric winding resistances $R_A = R_B = R_C$ and servo amplifier gains $K_{SA} = K_{SB} = K_{SC}$, the functional to be minimized can be written as

$$f = \sum_p u_p^2\,, \qquad p \in \{A, B, C\}. \qquad (15)$$

If the resistances are unsymmetric and/or the gains are unequal and known, (15) has to be adapted to meet the requirements. In case of small unsymmetric resistances and gains, the minimization of (15) minimizes the winding losses approximately. This assumption did not have any effect on the force ripple, only the winding losses may not be minimal, if the amplifier gains are unequal or the resistances are unsymmetric. After substituting (13) in (15) the current commands can be obtained by minimizing (15)

$$u_p = \frac{\partial f}{\partial u_p} = 0\,, \qquad p \in \{A, B, C\}, \tag{16}$$

as

$$u_p(u, x) = \frac{K_p\,(x)}{K_A^2\,(x) + K_B^2\,(x) + K_C^2\,(x)} K_F\, u. \tag{17}$$

The resulting equations depend only on the force functions $K_p(x)$ and on the force command. Since the force equations are usually not known, they have to be identified. The identification procedure is described in section IV.

B. Motors with Star-Connection

In motors with star-connection the constraint is represented by

$$F_{\text{thrust}} = K_A(x)\,u_A + K_B(x)\,u_B = K_F\,u \neq f(x). \tag{18}$$

Assumption of symmetric winding resistances $R_A = R_B = R_C$ and servo amplifier gains $K_{SA} = K_{SB}$ leads to

$$f = u_A^2 + u_B^2 + u_A\,u_B\,. \tag{19}$$

After substituting (18) in (19) the current commands can be obtained by minimizing (19)

$$u_p = \frac{\partial f}{\partial u_p} = 0\,, \qquad p \in \{A, B\}, \tag{20}$$

as

$$u_A(u, x) = \frac{K_A\,(x) - \frac{1}{2} K_B\,(x)}{K_A^2\,(x) + K_B^2\,(x) - K_A\,(x)\,K_B\,(x)} K_F\, u, \tag{21}$$
$$u_B(u, x) = \frac{K_B\,(x) - \frac{1}{2} K_A\,(x)}{K_A^2\,(x) + K_B^2\,(x) - K_A\,(x)\,K_B\,(x)} K_F\, u$$

The current commands depend on the unknown force functions $K_A(x)$ and $K_B(x)$ and on the force command u. The next section describes how to identify the force functions in the case of nonidealities.

IV. Identification of Force Functions

In an ideal servo system, the force functions are symmetric, balanced sinusoids. Nonidealities of the force functions as harmonics in shape, phase or amplitude imbalance lead to force ripple if sinusoidal commutation is applied. As described in section III an optimal commutation can prevent force ripple. For optimal commutation the force functions have to be known. This section describes a method for identification of force functions. The method consist of two steps, in the first step the force function $K_{F\sin}(x)$ for sinusoidal commutation is identified. In the second step the force functions for each current command is identified separately. Before identification can be performed, possible offset currents have to be compensated as described in [14].

A. Identification of $K_{F\sin}(x)$

The main idea of the proposed method is to identify $K_{F\sin}(x)$ in a closed position control loop, by storing the force command, which is the output of the position controller, at constant load force F_{load}, as a function of position $u_{\sin}(x)$. In order to avoid inaccuracy by stiction, the measurement is achieved with moving carriage. In the experimental setup, the constant load force is produced by the force of gravitation. At constant loads, the controller changes the force command over the position in order to compensate the variation of $K_{F\sin}(x)$. For constant load forces and sinusoidal commutation the force equation is given by

$$F_{\text{load}} = K_{F\sin}(x)\,u_{\sin}(x). \tag{22}$$

After measuring $u_{\sin}(x)$ at a given constant load force F_{load}, $K_{F\sin}(x)$ can be obtained as

$$K_{F\sin}(x) = \frac{F_{\text{load}}}{u_{\sin}(x)}. \tag{23}$$

The values of $K_{F\sin}(x)$ are stored at equidistant positions for the later use in identification of K_p.

B. Identification of $K_p(x)$

The force functions $K_p(x)$ describes the force generated by the current command u_p. In order to identify the force function $K_p(x)$ an offset o_p is added to the current command u_p. The proposed identification procedure measures the force command u_{op} over the position at a given offset o_p and sinusoidal commutation. The offset produces a force, that is proportional to the force function. The position controller changes the force command, to compensate this additional force. In order to identify the force function $K_A(x)$ an offset o_A is added to the current command u_A:

$$u_A(u, x) = \frac{2}{3}\,\sin(\vartheta(x))\;u + o_A. \tag{24}$$

Plugging (24) into (4) or (10) at sinusoidal commutation of the other phases reveals

$$F_{\text{load}} = K_{F\sin}(x)\,u_{oA}(x) + K_A(x)\,o_A. \tag{25}$$

After subtracting (22) from (25) the force equation is obtained as

$$K_A(x) = \frac{(u_{oA}(x) - u_{\sin}(x))\;K_{F\sin}(x)}{o_A}. \tag{26}$$

The other force functions can be identified in the same manner [19]. The choice of the right value for o_p is important for proper identification. If the value is to small, the noise prevents sufficient identification. The operation of the motor is disturbed, if the value is to high. Experiments show, that a value of 10% of the peak command should be a good compromise.

V. Experimental Results

A. Experimental Setup

The motor used in the setup is a PM LSM with epoxy core. There are two basic classifications of PM LSMs: epoxy core (i.e. non-ferrous, slotless) and iron core. Epoxy core motors have coils wound within epoxy support. These motors have a closed magnetic path through the gap since two magnetic plates "sandwich" the coil assembly [17]. The secondary induces a multipole magnetic field in the air gap between the magnetic plates. The magnet assembly consists of rare earth magnets, mounted in alternate polarity on the steel plates. The motor drives a mass of total 1.5 kg and is vertically mounted.

The servo amplifier employed in the setup is a PWM type with closed current control loop. The software commutation of the three phases is performed in the motion controller as described. The initialization routine for determining the phase relationship is part of the motion controller.

The maximum current commands u_A, u_B of the servo amplifier (± 10 V) correlate to the peak currents of the current loops (i_{ref}). In the setup the peak current of the amplifier is 20 A. This leads to an amplifier gain of $K_S = 2\,{}^{\text{A}}/_{\text{V}}$. The PWM works with a switching frequency of 20 kHz. The current loop bandwidth is specified with 3 kHz. [20]

B. Identification of $K_{F\sin}(x)$

Fig. 1 shows the identified force function $K_{F\sin}$ as a function of the position x and the spectrum of $K_{F\sin}$. The spectrum of $K_{F\sin}$ is calculated with least square estimation as described in [14]. The fundamental is chosen as the commutation period ($2\,\tau_p = 30$ mm). The maximum ripple is at the 2^{th} order harmonic.

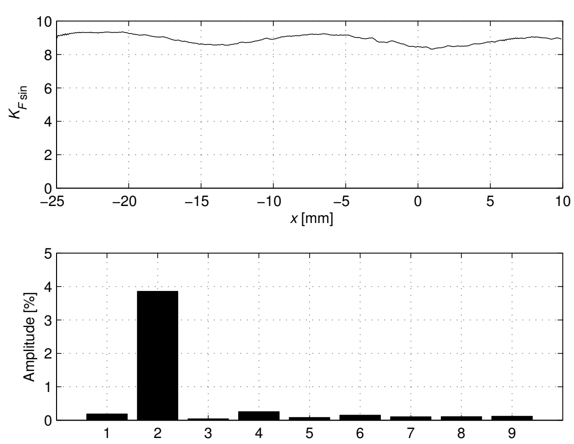

Fig. 1. Force function $K_F(x)$ with spectrum

C. Identification of K_A and K_B

Fig. 2 shows the identified force functions K_A and K_B. Identification is performed with different values for o_A and o_B. The mean values of all experiments are calculated in order to identify $K_A(x)$ and $K_B(x)$ properly. The curves show that there is a amplitude imbalance in the force functions. The imbalance causes force ripple of the second order harmonic.

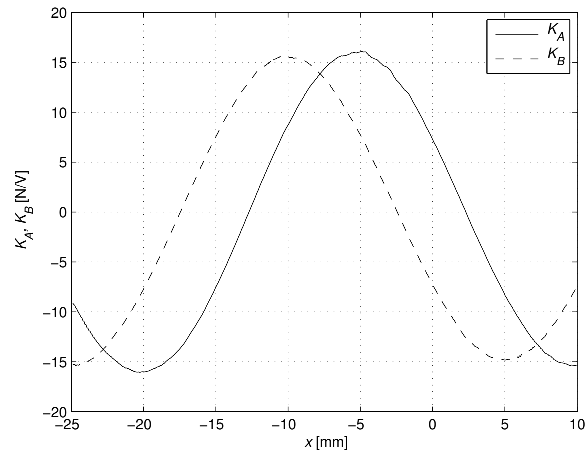

Fig. 2. Identified force functions

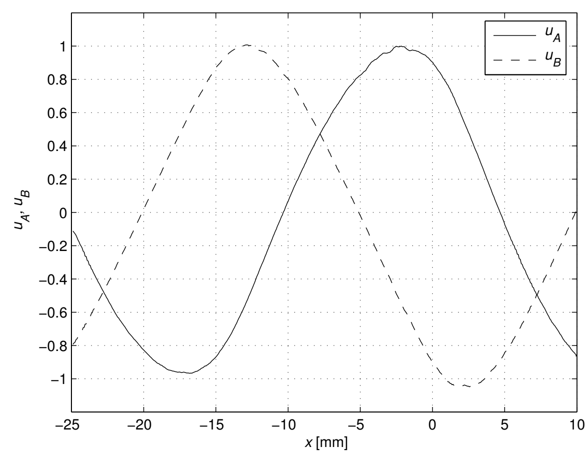

Fig. 3. Optimal current commands for constant force

Fig. 3 shows the optimal current commands over position at a constant force command. Fig. 4 shows the identified force function $K_{F\text{opt}}$ as a function of the position x and the spectrum of $K_{F\text{opt}}$. The force ripple is significantly reduced, when the optimal commutation law is applied.

VI. Controller Design

Fig. 5 shows the block diagram of the servo control system. In order to achieve a better tracking performance, a feedforward controller is applied. Feedback control without feedforward control always introduces a phase lag in the command response. Feedforward control sends an additional output, besides the feedback output, to drive the servo amplifier input to desired thrust force. The feedforward control

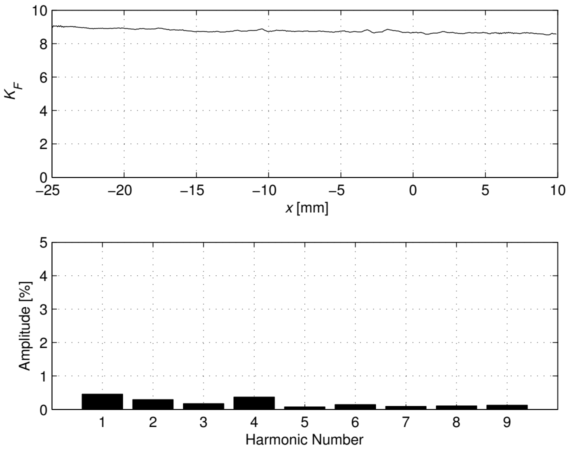

Fig. 4. $K_F(x)$ at optimal commutation

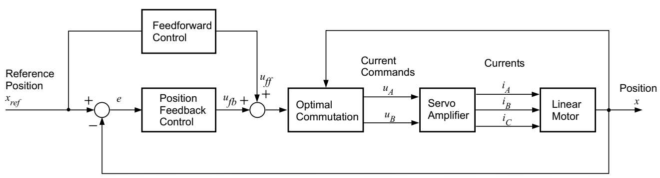

Fig. 5. Controller design

compensates the effect of the carriage mass and friction force. The friction force is modeled by a kinetic friction model and identified with experiments at different velocities. The mass of the carriage is identified with a dynamic least square algorithm.

If cogging force is significant the feedforward control may add a position dependent signal to compensate this disturbance force. The stability of the system is determined by the feedback loop (PD controller). Experiments show, that the tracking performance is improved significantly, if the optimal commutation law is applied. The performance is in the same order as described in [14]. The maximum position error is $\approx 30\%$ lower than without any compensation of force ripple.

VII. Conclusion

In this paper, an optimal commutation law for three-phase LSMs with surface mounted PMs is presented. The optimized commutation of the phase currents generates smooth force and produce minimal winding losses, which maximizes motor efficiency. The commutation law is valid for any velocity and any desired thrust force. It considers nonidealities of motor and amplifier. It is based on non-parametric force function, which are identified in a proposed procedure. No assumptions of periodicity, symmetry, shape or balance of the force functions are made. Experiments are performed with a epoxy-core PM LSM, but the results are also valid for rotating motors.

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