Identification of Force Functions for Permanent Magnet Linear Synchronous Motors

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Abstract Sinusoidal commutation of phase currents leads to force ripple, if motor back-EMFs differ from ideal case. Force ripple reduces the tracking performance significantly, if no compensation methods are applied. To overcome force ripple, optimal commutation of phase currents is proposed. Optimal commutated currents generate smooth force and produce minimal winding losses and therefore maximize motor efficiency. The optimal commutation law depends on the force functions produced by the phase currents. This paper presents a method for identification of non-parametric force functions of servo systems with linear motors. In this research, synchronous motors with surface-mounted magnets and three-phase star wiring are considered.

Keywords: force ripple, identification, optimal commutation law

1. Introduction

Permanent magnet (PM) linear synchronous motors (LSM) are mainly used in applications which require high speed and high precision in positioning such as semiconductor manufacturing or laser cutting. The more predominant nonlinear effects underlying a PM LSM system are friction and force ripple arising from imperfections in the underlying components [1]. There are two types of position dependent disturbances: cogging force and force ripple. Force ripple is an electro-magnetic effect and causes a periodic variation of the force constant. Only if the back-EMF waveforms are sinusoidal and balanced, symmetric sinusoidal commutation of the phase currents produces smooth force. Cogging force is a disturbance force that is independent of the motor currents.

In order to avoid force ripple, different methods have been developed. In [2] several techniques of torque ripple minimization for rotating motors are reviewed. In [3] a force ripple model is developed and identification is carried out with a force sensor and a frictionless air bearing support of the motor carriage. In [4] a neuronal-network based feedforward controller is proposed to reduce the effect of force ripple. Position-triggered repetitive control is presented in [5].

In [6], a force ripple compensation method for PM LSM systems with electronic commutated servo amplifiers was presented. The proposed method applies a feedforward controller for compensation of force ripple and friction. The drawback of controller-based compensation methods in conjunction with electronic sensor-based commutation is the non optimal current excitation of the motor. Imperfect commutation of the phase currents produces additional heat in the phase windings and therefore reduces the motor efficiency.

In [7], the author has presented a force ripple compensation method for software commutated servo amplifiers. The additional input signal of a software commutated amplifier is used to optimize the operation of the motor. In that paper a method for design of the phase current commands for minimization of force ripple and maximization of motor efficiency is presented. The commutation of the phase currents is optimized in order to get smooth force and minimal winding losses. The optimal commutation law depends on the force functions produced by the phases.

In section 4 of this paper, a method for identification of the force functions is presented. In section 2 a model of PM LSMs is developed and in section 3 the optimal commutation law is presented.

2. Modeling of Three-Phase Servo Systems

In this section a model of force generation for PM LSMs is developed. The electromagnetic thrust force is produced by the interaction between the PMs and the magnetic field in the windings driven by the phase currents of a servo amplifier. In PM LSMs, three different types of forces exists:

- *Cogging force:* Cogging is a magnetic disturbance force that is caused by attraction between the PMs and the iron part of the primary. The force depends on the relative position of the primary with respect to the magnets, and it is independent of the motor current. The mean value of cogging is zero. Cogging is negligible in motors with iron-less primaries or slotless motor design. If cogging is significant, a position dependent signal has to be added to the force command to overcome the cogging force. This can be made for example by means of feedforward control [4].
- *Reluctance fore:* Reluctance force occurs only in motors with interior-mounted PMs. In this type of motor, the reluctance of the motor is a function of position. The self inductance of the phase windings varies with position of the primary with respect to the secondary. When current flows, this causes a position dependent force. If PMs are surface-mounted, reluctance is constant and reluctance force is negligible. In this work reluctance force will not be considered.
- *Excitation fore:* Excitation force is produced by the interaction between the magnetic field in the secondary and the magnetic field of the phase windings. In Surface-

mounted PM motors, this is the dominant force production mechanism. In this paper only this type of force is considered.

Excitation force is proportional to the magnetic field and the phase currents i_A, i_B, i_C. The back-EMF induced in a phase winding (e_A, e_B, e_C) is proportional to the magnetic field and the speed of the motor. The total thrust force F_{thrust} is the sum of the forces produced by all phases:

$$\dot{x}\, F_{thrust} = \sum_p e_p(x)\, i_p\ , \qquad p \in \{A, B, C\}. \qquad (1)$$

The speed-normalized back-EMF waveforms $\frac{e_p(x)}{\dot{x}}$ can also be interpreted as the force functions of the phases ($K_{M_p}(x)$). With this force functions the thrust force can be rewritten as:

$$F_{thrust} = \sum_p K_{M_p}(x)\, i_p\ , \qquad p \in \{A, B, C\}. \qquad (2)$$

Only if the force functions are sinusoidal and balanced, symmetric sinusoidal commutation of the phase currents produces smooth force. Force ripple occurs, if the motor current is different from zero, and its absolute value depends on the required thrust force and the relative position of the primary with respect to the secondary. There are several sources of force ripple in motor and amplifier. Due to production tolerances, phase and amplitude of back-EMFs may be imbalanced. Furthermore the shape of back-EMFs may differ from ideal sinusoidal shape, due to motor design. Since an amplifier drives the phase currents, it may also be a source of force ripple. Production tolerances of the amplifier may lead to offset currents, imbalance of current gains and measurement gains.

Inspection of (2) shows, that offset currents lead to force ripple with the same period as the force functions. This force ripple is independent of the desired thrust force. Amplitude or phase imbalance of the motor and imbalance of amplifier gains lead to force ripple with half commutation period which scale in direct proportion to the desired thrust force. A k^{th} order harmonic of a back-EMF produces $(k-1)^{th}$ and $(k+1)^{th}$ order harmonic force ripple, if a sinusoidal current is applied.

Force ripple with the same period as the commutation period is independent of the desired current, all higher order harmonics scale in direct proportion to the desired current, because of the linearity of the force equation (2).

The star connection is the most common configuration in three-phase motors. The three external lines of the star are connected to a half bridge circuit. The collection of three half bridges is called a three-phase bridge. Only six power electronic devices are needed in a three-phase bridge, this is minimal compared to any other number of phases. Only two of the three phase currents are independent, the third phase current depends on the others:

$$i_C = -i_A - i_B. \qquad (3)$$

The phase currents are driven by a servo amplifier which needs only two command signals for the three phases:

$$\begin{aligned} &i_A = K_{S_A}\, u_A\ , \quad i_B = K_{S_B}\, u_B\ , \\ &i_C = -K_{S_A}\, u_A - K_{S_B}\, u_B, \end{aligned} \qquad (4)$$

where u_A and u_B are the current commands and K_{S_A} and K_{S_B} are the amplifier gains. The force equation can be written as

$$F_{thrust} = K_A(x)\, u_A + K_B(x)\, u_B, \qquad (5)$$

where $K_A(x)$ and $K_B(x)$ are position dependent force functions of the phases, which are given by

$$\begin{aligned} K_A(x) &= K_{S_A}\ (K_{M_A} - K_{M_C}) \quad \text{and} \\ K_B(x) &= K_{S_B}\ (K_{M_B} - K_{M_C})\,. \end{aligned} \qquad (6)$$

In sinusoidal commutation, the current commands are functions of the force command u and position x and are defined by

$$\begin{aligned} u_A(u,x) &= \frac{2}{3}\, \sin\left(\vartheta(x)\right)\, u, \\ u_B(u,x) &= \frac{2}{3}\, \sin\left(\vartheta(x) \pm \frac{2\,\pi}{3}\right)\, u, \\ \text{with} \quad \vartheta(x) &= \frac{\pi}{\tau_p}\,(x - x_0) \end{aligned} \qquad (7)$$

where τ_p is the pole pitch and x_0 is the zero position with maximum force. The sign in $\sin\left(\vartheta(x) \pm \frac{2\pi}{3}\right)$ depends on the wirering of the motor. In case of sinusoidal commutation, the force equation can be rewritten as

$$\begin{aligned} F_{thrust} &= K_{F_{sin}}(x)\, u \quad \text{with} \\ K_{F_{sin}}(x) &= \frac{2}{3}\Bigg(\sin\left(\vartheta(x)\right)\, K_A(x) + \\ &\qquad \sin\left(\vartheta(x) \pm \frac{2\,\pi}{3}\right)\, K_B(x) \Bigg). \end{aligned} \qquad (8)$$

Thrust force is only independent of position in the ideal case, where the force functions $K_A(x)$ and $K_B(x)$ are balanced sinusoids.

3. Optimal Commutation Law

Aim of optimal commutation is to obtain current commands that generate a thrust force which depends only on the force command and which is independent of the position. Inspection of the force equation (5) reveals, that there are an infinite number of combinations of current commands that generate the desired force. Therefore a secondary condition has to be applied. In this research, the ohmic winding losses of the motor are minimized. In permanent magnet synchronous motors, the winding losses are the main source of power dissipation.

The problem is formulated as constrained optimization, where the thrust force is the constraint and the losses are minimized. In motors with star-connection the constraint is represented by

$$F_{thrust} = K_A(x)\, u_A + K_B(x)\, u_B = K_F\, u \neq f(x). \quad (9)$$

where K_F is a freely eligible constant and u is the force command. The winding losses are given by

$$P_{cu}(x) = \sum_p R_p\, i_p^2(x)\ , \qquad p \in \{A, B, C\}, \qquad (10)$$

where R_p is the resistance of phase winding p. In assumption of symmetric winding resistances $R_A = R_B = R_C$ and servo amplifier gains $K_{S_A} = K_{S_B}$, the functional to be minimized can be written as

$$f = u_A^2 + u_B^2 + u_A\, u_B\,. \qquad (11)$$

After substituting (9) in (11) the current commands can be obtained by minimizing (11)

$$u_p = \frac{\partial f}{\partial u_p} = 0\,, \qquad p \in \{A, B\}, \qquad (12)$$

as

$$u_A(u,x) = \frac{K_A\,(x) - \frac{1}{2}K_B\,(x)}{K_A^2\,(x) + K_B^2\,(x) - K_A\,(x)\,K_B\,(x)} K_F\, u, \qquad (13)$$

$$u_B(u,x) = \frac{K_B\,(x) - \frac{1}{2}K_A\,(x)}{K_A^2\,(x) + K_B^2\,(x) - K_A\,(x)\,K_B\,(x)} K_F\, u$$

The current commands depend on the unknown force functions $K_A(x)$ and $K_B(x)$ and on the force command u. The next section describes how to identify the force functions in case of nonidealities.

4. Identification of Force Functions

The method consist of two steps, in the first step, the force function $K_{F_{sin}}(x)$ for sinusoidal commutation is identified. In the second step, the force functions for each current command is identified separately. Before identification can be performed, possible offset currents have to be compensated as described in [7].

4.1 Identification of $K_{F_{sin}}(x)$ The main idea of the proposed method is to identify $K_{F_{sin}}(x)$ in a closed position control loop, by storing the force command, which is the output of the position controller, at constant load force F_{load}, as a function of position $u_{sin}(x)$. In order to avoid inaccuracy by stiction, the measurement is achieved with moving carriage. In the experimental setup, the constant load force is produced by the force of gravitation. At constant loads, the controller changes the force command over position in order to compensate the variation of $K_{F_{sin}}(x)$. For constant load forces and sinusoidal commutation the force equation is given by

$$F_{load} = K_{F_{sin}}(x)\, u_{sin}(x). \qquad (14)$$

After measuring $u_{sin}(x)$ at a given constant load force F_{load}, $K_{F_{sin}}(x)$ can be obtained as

$$K_{F_{sin}}(x) = \frac{F_{load}}{u_{sin}(x)}. \qquad (15)$$

The values of $K_{F_{sin}}(x)$ are stored at equidistant positions for later use in identification of K_p.

4.2 Identification of $K_p(x)$ The force functions $K_p(x)$ describes the force generated by the current command u_p. In order to identify the force function $K_p(x)$ an offset o_p is added to the current command u_p. The proposed identification procedure stores the force command $u_{o_p}(x)$ over position at a given offset o_p and sinusoidal commutation. The offset produces a force, that is proportional to the force function. The position controller changes the force command, to compensate this additional force. The force function $K_A(x)$ is identified by adding an offset o_A to the current command u_A:

$$u_A(u,x) = \frac{2}{3}\,\sin\left(\vartheta(x)\right)\, u + o_A. \qquad (16)$$

Plugging (16) into (5) at sinusoidal commutation of the other phases reveals

$$F_{load} = K_{F_{sin}}(x)\, u_{o_A}(x) + K_A(x)\, o_A. \qquad (17)$$

After subtracting (14) from (17) the force equation is obtained as

$$K_A(x) = \frac{\left(u_{o_A}(x) - u_{sin}(x)\right)\, K_{F_{sin}}(x)}{o_A}. \qquad (18)$$

The force function $K_B(x)$ can be identified in the same manner. The choice of the right value for o_p is important for proper identification. If the value is to small, the noise prevents sufficient identification. The operation of the motor is disturbed, if the value is to high. Experiments show, that values $5\% \ldots 10\%$ of the peak command obtain good results.

5. Experimental Results

5.1 Identification of $K_{F_{sin}}(x)$ Fig. 1 shows the identified force function $K_{F_{sin}}$ as a function of the position x and the spectrum of $K_{F_{sin}}$. The identification is performed after offset compensation. The spectrum of $K_{F_{sin}}$ is calculated with least square estimation as described in [7]. The fundamental is chosen as the commutation period ($2\,\tau_p = 30\,mm$). The ripple has a maximum at the 2^{th} order harmonic.

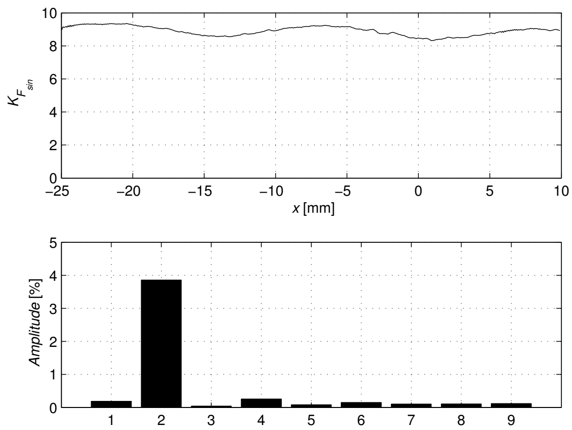

Fig. 1. Force function $K_F(x)$ with spectrum

5.2 Identification of K_A and K_B Fig. 2 shows the identified force functions K_A and K_B. Identification is performed with positive and negative values for o_A and o_B ($\pm\,0.5\,\mathrm{V}$). The mean values of the experiments are calculated in order to prevent inaccuracy by offsets. The curves show, that there is an amplitude imbalance in the force functions. The imbalance causes force ripple of the 2^{th} order harmonic as shown in Fig. 1. Fig. 3 shows the optimal cur-

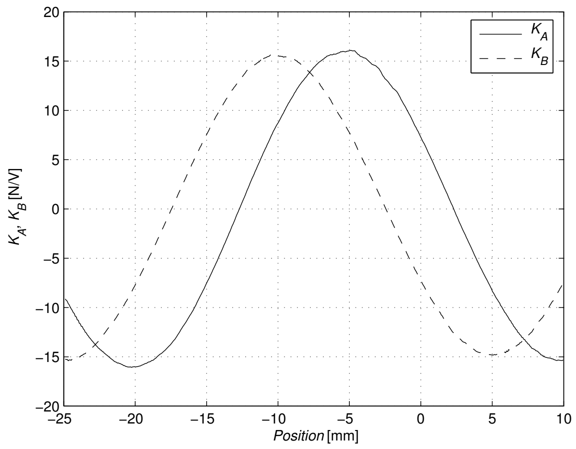

Fig. 2. Identified force functions

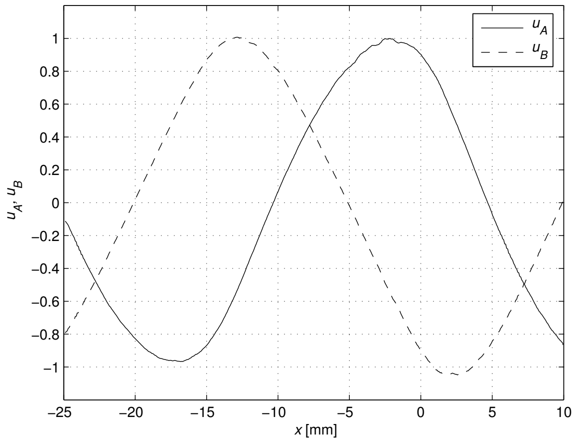

Fig. 3. Optimal current commands for constant force

rent commands over position at a constant force command. Fig. 4 shows K_F at optimal commutation. The ripple on K_F is significantly reduced compared to sinusoidal commutation.

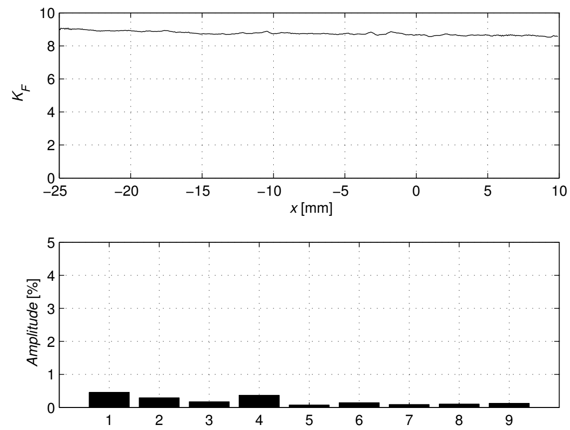

Fig. 4. $K_F(x)$ at optimal commutation

6. Controller Design

Fig. 5 shows the block diagram of the servo control system. In order to achieve a better tracking performance, a feedforward controller is applied. The feedforward control compensates the effect of the carriage mass and the friction force. The friction force is modeled by a kinetic friction model and identified with experiments at different velocities. The mass of the carriage is identified with a dynamic least square algorithm. If cogging force is significant the feedforward control may add a position dependent signal to compensate this disturbance force. The stability of the system is determined by the feedback loop. Experiments show, that the tracking performance is improved significantly, if the optimal commutation law is applied [7].

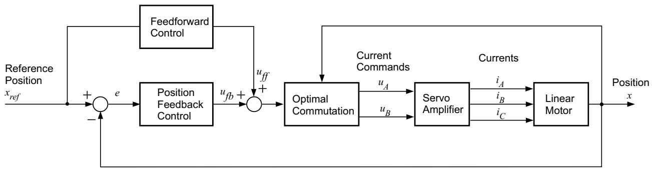

Fig. 5. Controller design

7. Conclusion

In this paper, the identification of force functions for an optimal commutation law is presented. The optimal commutation law guarantees smooth force and minimal winding losses. It is valid for any velocity and any desired thrust force and considers nonidealities of motor and amplifier. The optimal commutation law is based on non-parametric force function. The force functions are identified in a closed control loop, by measuring control signals over position at sinusoidal commutation. No assumptions of periodicity, symmetry, shape or balance of the force functions are made. Experiments are performed with an epoxy-core PM LSM, but the results are also valid for rotating motors.

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